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Benchmark of Emerging Structural Reliability Methods

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ABSTRACT: The present contribution provides a benchmark on a selection of emerging methods for assessing structural reliability. The benchmark includes the Polynomial Chaos Expansion, the Subset Simulation, the Probability Density Evolution Method, the Monte Carlo Simulation Extrapolation Technique and the Robust Importance Sampling technique. The benchmark is conducted for a range of different typical categories of structural reliability problems, including time invariant, time variant and dynamic systems. For each of the considered categories of reliability analysis first the applicability of the different reliability analysis methods is assessed and thereafter the efficiency of the applicable methods is assessed and compared. The benchmark includes reliability analyses of several principal structural reliability problems of relevance in practice.

1. INTRODUCTION

Over the past 40 – 50 years significant progress has been achieved on the development of modern methods of structural reliability. The literature on structural reliability analysis techniques has virtually exploded since the early day developments of First Order Reliability Methods (FORM) and Monte Carlo Simulation (MCS) techniques. The available selection of different formulations of techniques and algorithms is by now very rich and greatly supports reliability analysis in practice, in 1985 the number of publications found in *Web of Science* with the keywork *Structural Reliability Analysis* is 152, by the year 2000 is increased to 2,866 and by now has reached 20,707. Indeed modern methods of structural reliability are being applied on a daily basis both for the purpose of calibrating safety formats of modern semi-probabilistic design codes and, as a direct means, for supporting decisions on service life extensions, identification of strengthening options, and for optimization of inspection and maintenance activities, see e.g. JCSS (2001).

However, in practical application such as in e.g. reliability analysis of offshore structures subject to extreme environmental loads and degradation due to fatigue and subsidence, the efficient formulation and analysis of structural reliability still poses a substantial challenge. In such contexts the formulation of the reliability problems essentially have the form of first excursion problems where one must account for not only the non-linear and dynamic structural responses but also a rather involving representation of the environmental loads – with due account of the complex hierarchical dependencies between the various uncertainties affecting the structural performances.

In such cases, it is not immediately clear which of the available reliability analysis techniques is the most efficient and robust choice. In practice the first resort is often to apply Crude Monte Carlo Simulation (CMCS). Such a strategy might be feasible as a starting point to understand the characteristics of a specific reliability analysis problem but for repeated analyses, such an approach is in general not an option due to the associated very significant computational efforts.

The present paper reports on first results from a research project conducted as part of the Danish Hydrocarbon Research and Technology Center (DHRTC). The research project aims to provide clarity on choise of appropriate and efficient reliability analysis technique, for different typical reliability problems as these occur in the context of reliability based design, and assessments of existing steel jacket offshore structures. The first phase of this research project is directed on the assessment of the strengths and possible limitations associated with a selection of more recent - emerging - reliability analysis techniques, namely: Subset Simulation (SS), Monte Carlo Simulation Extrapolation Technique (MCSET), Probability Density Evolution Method (PDEM), Polynomial Chaos Expansion (PCE), Robust Importance Sampling (RIS).

The more classical reliability analysis techniques such as FORM/SORM, Response Surface and Importance Sampling based MCS are thus excluded in the present benchmark, however, it is the intention to come back to these techniques at a later time.

The present paper is structured such that first a rather abridged presentation of the considered reliability analysis techniques is first provided. Thereafter the different techniques are benchmarked on a selection of principally different classes of typical reliability problems as these occur in practice. On this basis the techniques are compared with respect to their adequacy and performances. Finally, a discussion of the insights gained is provided, and an outlook to future research concludes the paper.

2. METHODS USED

2.1. Subset Simulation (SS)

SS as introduced in Au and Beck (2001a) aims to enhance numerical efficiency by concentrating samples in subsets of the space of realizations of the random variables where failure is more likely. In this manner, the SS technique is similar to importance sampling, however with the significant difference that the method does not require prior knowledge regarding the most likely failure point(s). These are identified through a sequence of steps as presented subsequently. The event of failure, F occurs when realizations of the random variables belong to the failure domain which may be written as:

$$F_k = \bigcap_{i=1}^{k} F_i, \quad k = 1, \dots, m \tag{1}$$

where the sets F_i, $i = 1, 2, \dots, m$ are ordered such that $F = F_m \subset \dots \subset F_2 \subset F_1$ why the probability of failure P_F can be written as:

$$P_F = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i) \tag{2}$$

Even for very small values of P_F, by choosing appropriate intermediate failure events $\{F_i : i = 1, \dots, m-1\}$, the conditional failure probabilities expressed by Eq. (2) are sufficiently large to be efficiently estimated by means of CMCS. As described in Au and Beck (2001a), the estimation of the conditional failure probabilities is greatly facilitated by Markov Chain MCS.

2.2. Monte Carlo Simulation Extrapolation Technique (MCSET)

The MCSET suggested by Qin et al. (2012) aims to enhance numerical efficiency by sampling in a scaled space where the failure domain is expanded by a scaling factor – and the probability of failure correspondingly is enlarged. By repeating this process for a limited number of different scale factors the asymptotic integral expansion due to Laplace facilitates extrapolation of the calculated probabilities to the original unscaled space.

Characterizing the integration domain of the probability integral by a scaling factor λ as:

$$I(\lambda) = \int_{D(\lambda)} p(\mathbf{z})d\mathbf{z} = \int_{D} p(\lambda\mathbf{z})d\mathbf{z} \tag{3}$$

where $p(\mathbf{z})$ is the joint Probability Density Function (PDF) of the standard Normal distributed random variables $\mathbf{Z} = (Z_1, Z_2, \dots, Z_n)^T$ and the domain $D(\lambda)$ is the integration domain D scaled by a real valued positive scalar $\lambda \in [0, 1]$. The integral in Eq. (3) may also be written in the following form:

$$I(\lambda) = \lambda^n \int_{D} \frac{1}{(2\pi)^{n/2}} e^{-\frac{\lambda^2}{2}||\mathbf{z}||^2} d\mathbf{z} \tag{4}$$

This integral has a form for which the asymptotic solution due to Laplace applies. Based on this solution a functional form of the extrapolation is proposed by Qin et al. (2012) as:

$$I(\lambda|\mathbf{q}) = a_\infty \frac{1-e^{-b_1-b_2\lambda^2}}{1-e^{-b_3-b_4\lambda^2}} \prod_{i=1}^{k} \Phi(-c_i^*\lambda) \qquad (5)$$

where $\mathbf{q} = (a_\infty, b_1, b_2, b_3, b_4, c_1, \ldots, c_k)^T$ is a vector whose components are estimated by means of regression analysis (see Qin et al. (2012) for more details) and $I(\lambda|\mathbf{q})$ corresponds to the probability integral for the original domain scaled by λ, these values can be calculated using CMCS. The estimator for the failure probability P_F is $\widehat{P}_F = I(1|\mathbf{q}^*)$, where $\mathbf{q}^*$ is the vector which minimizes the sum of the squared residuals between the left and right side of the Eq. (5).

2.3. *Probability Density Evolution Method (PDEM)*

The PDEM proposed by Chen and Li (2005) facilitates probabilistic response and reliability analysis of dynamic systems, by providing the PDF of the considered system responses and their evolution over time. The method builds on the principle of probability preservation Chen and Li (2005). For the case of one failure mode, the reliability of the random dynamic system can be written as:

$$R(t) = P\{X(t) \in \Omega_S, \quad t \in [0,T]\} \qquad (6)$$

where Ω_S is the safe domain, and $X(t)$ the system response of interest. The Generalized Density Evolution Equation (GDEE) for Probability Dissipating systems, see e.g. Xu and Li (2016), is represented through the GDEE with an absorbing boundary condition such that:

$$\frac{\partial p_{X\mathbf{Z}}(x,\mathbf{z},t)}{\partial t} + \dot{X}(\mathbf{z},t)\frac{\partial p_{X\mathbf{Z}}(x,\mathbf{z},t)}{\partial x} = 0 \qquad (7)$$

$$p_{X\mathbf{Z}}(x,\mathbf{z},t) = 0, \quad X(\mathbf{z},t) \notin \Omega_S \qquad (8)$$

where $p_{X\mathbf{Z}}(x,\mathbf{z},t)$ denotes the joint PDF of $(X(t),\mathbf{Z})$ and $\mathbf{Z}$ is a vector of random variables representing the uncertainty in the physical parameters of the dynamic system and its excitation. The initial condition of Eq. (7) reads:

$$p_{X\mathbf{Z}}(x,\mathbf{z},t)|_{t=0} = \delta(x-x_0)p_{\mathbf{Z}}(\mathbf{z}) \qquad (9)$$

where x_0 is the deterministic initial value of x at $t=0$, $\delta(\cdot)$ is the Dirac function and $p_{\mathbf{Z}}(\mathbf{z})$ is the PDF of $\mathbf{Z}$, the remaining (or non-dissipated) PDF may thus be written as:

$$\widetilde{p}_X(x,t) = \int_{\Omega_{\mathbf{Z}}} p_{X\mathbf{Z}}(x,\mathbf{z},t)\mathrm{d}\mathbf{z} \qquad (10)$$

where Ω_Z represent the domain of $\mathbf{Z}$. Performing the integration over $\widetilde{p}_X(x,t)$ results in the reliability $R(t)$ of the random dynamic system:

$$R(t) = \int_{-\infty}^{\infty} \widetilde{p}_X(x,t)\mathrm{dx} \qquad (11)$$

2.4. *Polynomial Chaos Expansions (PCE's)*

PCE's comprise surrogate representations of probabilistic systems in terms of a set of coefficients in a basis commonly referred to as polynomial chaos. The PCE's may be applied in support of reliability analysis of complex random mechanical systems or for analyzing statistical moments of their response characteristics. When the PCE's have been established, the numerical efforts necessary to analyze PCE's are orders of magnitude smaller than those required to analyze the original systems. For the purpose of simplification of presentation the PCE is outlined in the following for the case where the input is a scalar valued random variable variable Z. In this case the random response Y of a model $M(Z)$ is given by:

$$Y = M(Z) \qquad (12)$$

The PDF of Z is $f_Z(z)$, $z \in \mathscr{D}_Z$, where $\mathscr{D}_Z$ is the sample space of Z. For any two functions $\phi_1, \phi_2 : z \in \mathscr{D}_Z \to \mathscr{R}$ it is possible to define the functional inner product:

$$< \phi_1, \phi_2 >= \int_{\mathscr{D}_Z} \phi_1(z)\phi_2(z) f_z(z) dz \qquad (13)$$

Where the right hand side of Eq. (13) corresponds to the expected value $E[\phi_1(Z)\phi_2(Z)]$ with respect to the random variable Z. Two functions are said to be orthogonal with respect to the probability measure if $E[\phi_1(Z)\phi_2(Z)] = 0$. Then it is possible to build a family of orthonormal polynomials $\{\psi_k, k \in \mathcal{N}\}$ satisfying $< \psi_j, \psi_k >= \delta_{jk}$, where the subscript k denotes the degree of the polynomial ψ_k and δ_{jk} is the Kronecker delta equal to 1 when $k = j$ and 0 otherwise. Then the model response Y is approximated with a PCE truncated to the finite order M as:

$$Y \approx \sum_{i=0}^{M} u_i \psi_i(Z) \tag{14}$$

in which u_i are the polynomial coefficients and $\psi_i(z)$ the polynomial chaos basis. The formulation provided in the forgoing is readily extended to multivariate polynomials, where the input of the model is a vector of random variables, see e.g. Sudret (2015).

2.5. *Robust Importance Sampling (RIS)*

The RIS technique proposed by Valdebenito et al. (2014) aims to estimate the first excursion probability of random linear dynamic systems under Normal distributed excitations. The approach is based on the strategy presented in Au and Beck (2001b), where an Importance Sampling Density (ISD) function associated with the uncertain excitation is constructed for fixed structural system parameters, and on this basis an ISD function associated with the uncertainty in structural system parameters is introduced. For reasons of brevity the derivation of the ISD function are not provided here, and the reader is referred to Valdebenito et al. (2014) for more details.

3. BENCHMARK PROBLEMS

3.1. *General Remarks*

In this section, examples are provided with the objective to assess the adequacy and efficiency of each of the techniques introduced in the foregoing. To this end, three different reliability problems of practical relevance in probabilistic assessments of structural responses are considered. For each of these, first an assessment of the applicability of the different reliability analysis methods is given and thereafter, for the applicable methods, their efficiency is benchmarked. The efficiency is reported in the form of tables containing four main descriptors for each considered method. The first and the second columns contain the mean value and the coefficient of variation of the estimator for the failure probability $\widehat{P}_F$, respectively, both values are calculated as sample means established on the basis of 100 independent calculations for each reliability analysis method. The third column contains the total number N_s of evaluations of the structural responses for each calculation. The fourth column contains the *unit coefficient of variation* defined as:

$$\Delta = CV(\widehat{P}_F) \cdot \sqrt{N_s} \tag{15}$$

where $CV(\cdot)$ is the coefficient of variation. This value aims to compare the variance reduction achieved by the procedure, smaller values indicates a higher reduction, see e.g. Schueller and Pradlwarter (2007). In the case of the PDEM, due to the fact that samples are generated from a Sobol sequence, there is no variability in the results, why for this reliability analysis method Δ is not provided.

The first problem corresponds to a time invariant geotechnical reliability problem, the second to a time variant dynamical reliability problem with two excitation models: a) deterministic and b) Gaussian random process; the third problem is a time variant reliability problem with a degradation model.

The PCE does not readily facilitate analysis of time variant reliability problems without being combined with other techniques. The PDEM with the current formulation, has difficulties with reliability problems of high dimension. The RIS, as mentioned before, is only applicable in the specific case of linear dynamic systems under Normal distributed excitations. Due to these restrictions, only some of the considered reliability analysis techniques are applicable to the considered principal reliability problems. In Table 1 the symbol $\odot$ indicates that the technique is applicable, and the symbol $\otimes$ indicates that the method is omitted.

3.2. *Problem 1 - Geotechnical reliability problem*

This example considers the reliability of strip foundation, taken from Sudret (2015). The width of the strip foundation is B and the depth is D. The

Table 1: Benchmark problems and techniques used

Method:	SS	MCSET	PDEM	PCE	RIS
Problem 1	$\odot$	$\odot$	$\otimes$	$\odot$	$\otimes$
Problem 2 - a	$\odot$	$\odot$	$\odot$	$\otimes$	$\odot$
Problem 2 - b	$\odot$	$\odot$	$\odot$	$\otimes$	$\odot$
Problem 3	$\odot$	$\odot$	$\otimes$	$\otimes$	$\otimes$

soil layer is assumed homogeneous with cohesion c, friction angle ϕ and unit weight γ. The ultimate bearing capacity is modelled as:

$$q_u = cN_c + \gamma d N_q + \frac{1}{2} B\gamma N_\gamma \tag{16}$$

where the Load Bearing Capacity (LBC) factors are given in Table 2. The soil parameters and the foundation depth are modelled as independent random variables with probabilistic models summarized in Table 3. Collecting the random variables in the vector $\mathbf{Z}$ the random LBC of the strip foundation may be written as $q_u(\mathbf{Z})$.

Table 2: Bearing capacity factors - Problem 1

N_q	$e^{\pi \tan\phi} \tan^2(\pi/4 + \phi/2)$
N_c	$(N_q - 1)\cot\phi$
N_γ	$2(N_q - 1)\tan\phi$

Table 3: Probabilistic model of the random variables - Problem 1

Parameter	Distribution	Mean value	CV
B	Deterministic	10	-
d	Gaussian	1	0.15
γ	Lognormal	20	0.10
c	Lognormal	20	0.25
ϕ	Beta	Range:$[0,45]^\circ$, $\mu = 30^\circ$	0.10

Using the mean values of the variables in Table 3 the ultimate LBC is $\bar{q}_u = 2.98$ MPa, and the global safety factor where $SF = 4.0$. The reliability of the foundation with respect to its ultimate LBC is assessed through the Limit State Function (LSF) $g(\mathbf{z})$:

$$g(\mathbf{z}) = 1 - \frac{\bar{q}_u}{SF\, q_u(\mathbf{z})} \tag{17}$$

The results of the analysis results are shown in Table 4. The total number of samples N_s required using PCE, are those required to establish the surrogate model. From Table 4 it is seen that the PCE technique is the most efficient followed by the MCSET, the SS and finally CMCS.

Table 4: Results - Problem 1

Method	$\widehat{P}_F/10^{-4}$	$CV(\widehat{P}_F)$	N_s	Δ
CMCS	5.6	0.09	2.2×10^5	43
SS	5.7	0.08	3.7×10^4	15
MCSET	5.5	0.02	4.9×10^4	5
PCE	5.8	0.05	1.0×10^3	1

3.3. Problem 2 - linear oscillator

In this example, a linear Single Degree Of Freedom (SDOF) oscillator with natural frequency ω_n and critical damping d is considered. Both system parameters are modelled as independent Lognormal distributed random variables with mean values equal to π and 0.03 respectively, and both with a coefficient of variation equal to 0.1. The failure event of the considered system is defined as the first excursion of the oscillation x above the threshold x^*, why the LSF $g(\mathbf{r},\mathbf{z})$ may be written as:

$$g(\mathbf{r},\mathbf{z}) = 1 - \frac{\max_{t_k} |x(t_k,\mathbf{r},\mathbf{z})|}{x^*} \tag{18}$$

where $x(t_k,\mathbf{r},\mathbf{z})$ is the oscillation at time t_k, and where the vectors $\mathbf{R}$ and $\mathbf{Z}$ contain the random variables representing the uncertain system parameters and the uncertain excitation model variables, respectively.

3.3.1. Case a - Deterministic excitation

In the first case the system excitation is modelled as a product of a deterministic time dependent function, corresponding to an observed record of earthquake accelerations (Chile, 2010), and a random scale factor a_0 which is assumed to be standard Normal distributed, therefore $\mathbf{Z} = a_0$ and the threshold is equal to $x^* = 1$.

The results are shown in Table 5. In this case the PDEM is found to be most efficient. Figure 1 shows the estimates of the failure probability obtained for different thresholds. For relative large failure probabilities, the PDEM exhibits good accuracy, however, in the case of moderate to small probabilities ($< 10^{-3}$), the PDEM results appear to be biased. MCSET, SS and RIS all exhibit good accuracies for any threshold level, and the RIS is found to be more efficient than both MCSET and SS.

Table 5: Results - Problem 2 - case a

Method	$\widehat{P}_F/10^{-3}$	$CV(\widehat{P}_F)$	N_s	Δ
CMCS	2.2	0.07	1.0×10^5	21
SS	2.2	0.10	1.1×10^4	10
MCSET	2.3	0.09	6.3×10^4	24
RIS	2.2	0.10	6.0×10^3	1
PDEM	2.1	-	1.0×10^3	-

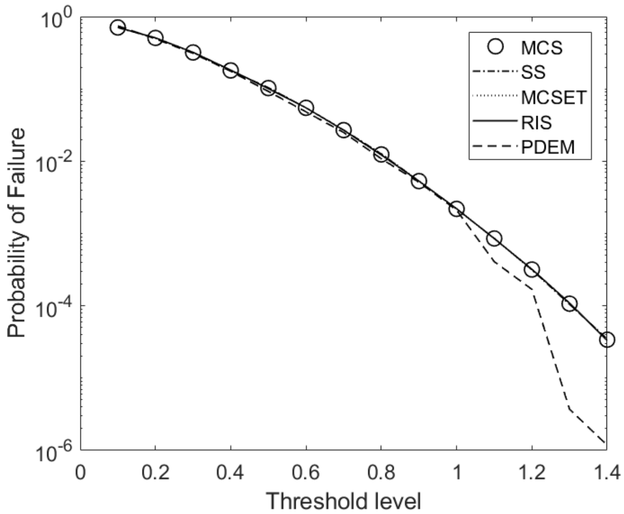

Figure 1: Results - Problem 2 - case a

3.3.2. *Case b - random excitation*

In case b) the system excitation is modelled by a random process in accordance with Li et al. (2009), where the auto-correlation function of the ground displacement process is obtained using the Wiener-Khintchine theorem as the inverse Fourier transform of the displacement spectrum. Thereby it is possible to obtain the correlation matrix R using the Hartley orthogonal base for which the orthogonal expansion is expressed as:

$$u(z,t) = \sqrt{2S_0}\sum_{j=1}^{M}\sqrt{\lambda_j}\xi_j(z_j)F_j(t) \tag{19}$$

where M is the truncation order, S_0 is the unit spectral intensity factor, λ_j are the eigenvalues of the correlation matrix R, and $F_j(t)$ are deterministic functions described in Li et al. (2009). The loading is represented through ten independent Normal distributed random variables $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_{10})^T$. In this case the threshold is set at $x^* = 0.53$.

The results are shown in Table 6. The PDEM has a significant bias for small probabilities. The RIS again exhibits a high efficiency compared to both MCSET and SS.

Table 6: Results - Problem 2 - case b

Method	$\widehat{P}_F/10^{-5}$	$CV(\widehat{P}_F)$	N_s	Δ
CMCS	1.1	0.09	7.0×10^6	271
SS	1.1	0.13	5.0×10^4	30
MCSET	1.1	0.10	4.2×10^5	65
RIS	1.1	0.09	6.0×10^3	7
PDEM	0.0	-	2.0×10^3	-

3.4. *Problem 3 - deteriorating structure subject to extreme loading*

In this example a structural system subject to fatigue degradation and ultimate failure caused by an extreme load event is considered. The time variant resistance $r(t)$ is modelled as:

$$r(t) = r_0\frac{a_{cr} - a(t)}{a_{cr} - a_0} \tag{20}$$

where r_0 is the initial resistance, a_0 and a_{cr} are the initial and critical crack lengths respectively. The crack length at time t, $a(t)$ is modelled as:

$$a(t) = \left(a_0^{\frac{2-m}{2}} + \frac{2-m}{2}\pi^{\frac{m}{2}}y^m c^m \sum_{i=1}^{N(t)} S_i^m\right)^{\frac{2-m}{2}} \tag{21}$$

where the parameters m, y and c represent the effects of geometry and material characteristics in the evolution of crack growth. S_i is the stress range of stress cycle i and $N(t)$ is the total number of cycles up to time t.

The parameters r_0, a_0, y and c are modelled by random variables contained in the vector $\mathbf{R}$. The probabilistic model of the random variables is summarized in Table 7.

Table 7: Probabilistic model of the random variables - Problem 3

Parameter	Distribution	Mean value	CV
m	Deterministic	3	-
a_{cr}	Deterministic	20	-
r_0	Lognormal	100	0.05
a_0	Lognormal	0.4	0.66
y	Lognormal	1	0.10
$\log c$	Normal	-25.5	0.03

In this example it is assumed that the stresses originate from time varying wind pressure modelled for 10 minute intervals as:

$$V(t,\mathbf{z}) = \bar{v}_{10} + u_f v(t,z^v) \tag{22}$$

where $\mathbf{Z} = (\overline{V}_{10}, U_f, Z^v)^T$, $\bar{v}_{10}$ is the mean wind speed at 10 m above the ground, U_f is the homogeneous random field and $v(t,z^v)$ is the wind velocity fluctuation process defined by the variable z^v.

The random process representing the wind pressure is modelled in accordance with Liu et al. (2016). To calculate the annual probability of failure it is necessary to simulate 10 storms of 6 hours duration, implying a total of 60 hours, in which a total of 360 sequences of realizations of random processes are merged involving 1080 random variables. The probabilistic model of the random variables is summarized in Table 8.

Table 8: Probabilistic model of the random variables - Problem 3

Parameter	Distribution	Distribution parameter	
$\bar{v}_{10}$	Weibull	(k,σ)	$=(2,6.0)$
u_f	Normal	(μ,σ)	$=(0,0.5)$
z^v	Normal	(μ,σ)	$=(0,1.0)$

The response of the structural system is modelled through a linear SDOF oscillator with natural frequency $\omega_n = \pi$ and critical damping $d = 0.03$. The oscillator is assumed loaded by the random process modelling the wind pressure described previously. The stress cycles are calculated from the response characteristics of the oscillator using Rainflow counting.

Failure of the system is defined as the first event of the structural response exceeding the ultimate capacity of the structure. The LSF is defined as:

$$g(t,\mathbf{r},\mathbf{z}) = 1 - \frac{S(t,\mathbf{z})}{r_0}\frac{a_{cr}-a_0}{a_{cr}-a(t,\mathbf{r},\mathbf{z})} \tag{23}$$

where $S(t,\mathbf{z})$ corresponds to the stress at time t. The results are shown in Table 9. In this example it is seen that SS is most efficient.

Table 9: Results - Problem 3

Method	$\widehat{P}_F/10^{-4}$	$CV(\widehat{P}_F)$	N_s	Δ
CMCS	1.20	0.30	1.0×10^5	95
SS	1.14	0.30	4.0×10^3	19
MCSET	1.34	0.35	6.0×10^3	27

4. CONCLUSIONS

To support the assessment and comparison of the different reliability analysis techniques considered in the forgoing basis is taken in the scheme proposed by Schueller and Pradlwarter (2007), see Table 10. The results are shown in Table 11, in where is indicated the performance of the technique relative to these properties.

Table 10: Definition of each Property Number (PN), adapted from Schueller and Pradlwarter (2007).

PN	Description
(1)	Applicable to any structural response
(2)	Ability to treat system parameter uncertainties
(3)	Applicable for stochastic excitation
(4)	Restriction on probabilistic dimensionality
(5)	Potential for further development
(6)	Treatment like black box
(7)	Efficiency
(8)	Implementation complexity
(9)	Applicable for any value of probability

As seen in Table 11, for the considered reliability analysis techniques it is not possible to identify an

Table 11: Characteristics of the proposed reliability estimation procedures

PN	MCS	SS	MCSET	RIS	PDEM	PCE
(1)	High	High	High	Low	Low	Low
(2)	High	High	High	Med	Med	High
(3)	High	High	High	High	High	Low
(4)	Low	Low	Low	Med	High	Low
(5)	Low	Low	High	Low	High	High
(6)	High	High	High	Low	Low	Med
(7)	Low	Med	Med	High	High	Med
(8)	Low	Low	Low	Med	High	Med
(9)	High	High	High	High	Low	Med

Med: Medium

efficient general method, which applies to all reliability problems; the efficiency and applicability of each technique is closely related to the problem on hand.

MCS based methods, such as SS and MCSET, exhibit good performances in estimating small failure probabilities for the three classes. Moreover, they facilitate analysis of dynamic random systems of high dimensionality.

The accuracy of SS relies on the expectation that the important area near the design point has been identified. For MCSET, the efficiency depends strongly on the set of λ values chosen, and the identification of the number of active constrains.

The three other studied techniques, even though they are restricted to certain classes of problems, exhibit outstanding efficiencies compared to SS and MCSET.

Future work will be directed on assessing how the different reliability analysis techniques may be optimized and combined in order to maximize robustness and efficiency in the analysis of different categories of reliability problems. Moreover, the insights gained will be included in the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS), where a guideline on the choice of reliability techniques for different categories of reliability problems is under preparation.

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